

# Mass Splitting of Three Seesaw Neutrinos

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## Abstract

If a family symmetry exists so that all three neutrinos have equal Majorana masses (via the seesaw mechanism), then the breaking of this symmetry from charged-lepton masses (via two-loop double  $W$  exchange) implies  $\Delta m$  less than about  $10^{-9} m_0$ . With the common mass  $m_0 \sim \text{eV}$  for hot dark matter,  $\Delta m^2 \sim 10^{-10} \text{ eV}^2$  is natural for vacuum solar neutrino oscillations.

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# 1 Introduction

The excitement generated by the Super-Kamiokande evidence of atmospheric neutrino oscillations [1] has prompted a great deal of theoretical activity in building models which explain it, as well as the possible evidence of solar neutrino oscillations [2]. With three known neutrinos, it is difficult to accommodate the LSND (Liquid Scintillator Neutrino Detector) results [3] as well, without sacrificing some pieces of the experimental data. Reviews of the overall situation are given in various other talks of these Proceedings. I will focus on the possibility of three nearly mass-degenerate neutrinos.

## 2 Common Mass for Three Neutrinos

Since neutrino oscillations only probe the difference of mass-squares, it is entirely possible that the three known neutrinos have a common large mass, but their splittings are small for some reason. This idea has received a lot of attention in the recent literature [4] and is being pursued actively at present [5].

A particularly desirable  $(\nu_e, \nu_\mu, \nu_\tau)$  mass matrix for oscillations, dark matter [6], and the absence of neutrinoless double beta decay [7] is given by

$$\mathcal{M} \simeq \begin{bmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix} m_0, \quad (1)$$

where the zero  $\nu_e$  diagonal entry ensures the absence of neutrinoless double beta decay. The eigenvalues of  $\mathcal{M}$  are  $-m_0$ ,  $m_0$ , and  $m_0$ , and the mass eigenstates are related to the interaction eigenstates by

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} \simeq \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/2 & -1/2 & -1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}. \quad (2)$$

Most model builders now add splitting so that

$$(\Delta m^2)_{13} \simeq (\Delta m^2)_{23} \sim 10^{-3} \text{ eV}^2 \quad (3)$$

to obtain  $\nu_\mu \rightarrow \nu_\tau$  oscillations with  $\sin^2 2\theta = 1$ . Further splitting is then added so that

$$(\Delta m^2)_{12} \sim 10^{-10} \text{ eV}^2 \quad (4)$$

to obtain  $\nu_e \rightarrow (\nu_\mu + \nu_\tau)/\sqrt{2}$  oscillations with  $\sin^2 2\theta = 1$  also. Note that  $m_0 \sim \text{eV}$  rules out the possibility of the small-angle matter-enhanced solution of the solar neutrino deficit in this context.

### 3 Natural Splitting of Three Seesaw Neutrinos

Whatever mechanism is assumed to obtain three mass-degenerate neutrinos, it is broken explicitly by charged-lepton masses. Hence there must be splitting among them from the physics of the standard model, supplemented only by the above-mentioned mass-generating mechanism. Consider thus the simplest such extension of the standard model. Add three heavy right-handed neutrino singlets  $N_{iR}$  as usual. Assume that they and the left-handed lepton doublets  $(\nu_i, l_i)_L$ , with  $i = +, 0, -$ , form  $SO(3)$  triplets. This yields the following invariant terms:

$$f[(\bar{\nu}_+ N_+ + \bar{\nu}_0 N_0 + \bar{\nu}_- N_-)\bar{\phi}^0 - (\bar{l}_+ N_+ + \bar{l}_0 N_0 + \bar{l}_- N_-)\phi^-], \quad (5)$$

and

$$M(2N_+ N_- - N_0 N_0). \quad (6)$$

In the basis  $(\nu_+, \nu_-, \nu_0, N_+, N_-, N_0)$ , the mass matrix is then

$$\mathcal{M}_{\nu, N} = \begin{bmatrix} 0 & 0 & 0 & m_D & 0 & 0 \\ 0 & 0 & 0 & 0 & m_D & 0 \\ 0 & 0 & 0 & 0 & 0 & m_D \\ m_D & 0 & 0 & 0 & M & 0 \\ 0 & m_D & 0 & M & 0 & 0 \\ 0 & 0 & m_D & 0 & 0 & -M \end{bmatrix}, \quad (7)$$

where  $m_D = f\langle\phi^0\rangle$ . Let  $m_D \ll M$ , the well-known seesaw mechanism [8] then reduces the above to a  $3 \times 3$  mass matrix for  $(\nu_+, \nu_-, \nu_0)$ :

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & -m_0 & 0 \\ -m_0 & 0 & 0 \\ 0 & 0 & m_0 \end{bmatrix}, \quad (8)$$

where  $m_0 = m_D^2/M$ .

Choose  $l_+ = e$ , then in general  $l_- = c\mu + s\tau$  and  $l_0 = c\tau - s\mu$ , where of course  $c = \cos\theta$  and  $s = \sin\theta$ . The degeneracy of  $\mathcal{M}_\nu$  is now lifted through the two-loop diagram [9] of Fig. 1. The result is [10]

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & -m_0 - s^2 I & -scI \\ -m_0 - s^2 I & 0 & scI \\ -scI & scI & m_0 + 2c^2 I \end{bmatrix}, \quad (9)$$

where

$$I = \frac{g^4}{256\pi^4} \frac{m_\tau^2}{m_W^2} \left( \frac{\pi^2}{6} - \frac{1}{2} \right) m_0 = 3.6 \times 10^{-9} m_0. \quad (10)$$

The new eigenvalues are  $-m_0 - s^2 I$ ,  $m_0$ , and  $m_0 + (1 + c^2)I$ . For small  $s^2$ , the above yields  $\nu_e \rightarrow \nu_\mu$  oscillations with  $\sin^2 2\theta \simeq 1$  and  $\Delta m^2 = 2s^2 m_0 I$  which is about  $3 \times 10^{-10} \text{ eV}^2$  for  $s = 0.1$  and  $m_0 = 2 \text{ eV}$ . This shows that vacuum solar neutrino oscillations are natural with three seesaw neutrinos of the same tree-level mass.

## 4 Atmospheric Neutrino Oscillations

It has been shown in the above that with equal tree-level seesaw masses for the three Majorana neutrinos, there is an irreducible splitting among them which is of the right magnitude [11] for vacuum solar neutrino oscillations. However, there is no explanation of atmospheric neutrino oscillations in this minimal version of the model. An *ad hoc* assumption may be made that the state  $c'\nu_0 + s'(\nu_+ - \nu_-)/\sqrt{2}$  acquires a mass  $m_1$  with  $m_1/m_0 \simeq 5 \times 10^{-4}$ . In that case, with  $s' \ll 1$  but not zero, the mass eigenvalues become  $-m_0 - s^2 I$ ,  $m_0 + (s^2 + 2\sqrt{2}s'sc)I$ , and  $m_0 + m_1$ , with

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} \simeq \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & s'/\sqrt{2} \\ c/\sqrt{2} & -c/\sqrt{2} + s's & -s - s'c/\sqrt{2} \\ s/\sqrt{2} & -s/\sqrt{2} - s'c & c - s's/\sqrt{2} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}. \quad (11)$$

Atmospheric  $\nu_\mu \rightarrow \nu_\tau$  oscillations now occur with  $\sin^2 2\theta = 4s^2c^2$  and  $\Delta m^2 = 2m_0m_1 \simeq 4 \times 10^{-3} \text{ eV}^2$ , and solar  $\nu_e \rightarrow c\nu_\mu + s\nu_\tau$  vacuum oscillations occur with  $\sin^2 2\theta \simeq 1$  and  $\Delta m^2 \simeq 4\sqrt{2}scs'm_0I \simeq 4 \times 10^{-10} \text{ eV}^2$ , if  $m_0 = 2 \text{ eV}$ ,  $s = c = 1/\sqrt{2}$ , and  $s' = 0.01$ .

## 5 Conclusion

The idea of nearly mass-degenerate neutrinos [4, 5] of a few eV may be cosmologically relevant as a component of dark matter [6]. If the origin of their common mass  $m_0$  is the seesaw mechanism [8], then there is an irreducible splitting [10] of the order  $10^{-9} m_0$  due to the charged-lepton masses. This is very suitable for vacuum solar neutrino oscillations with  $\Delta m^2 \sim 10^{-10} \text{ eV}^2$ . On the other hand, the above generic statement says nothing about atmospheric neutrino oscillations, but once the latter are incorporated (by hand or with the help of some new physics), then the residual splitting is available for solar neutrino oscillations.

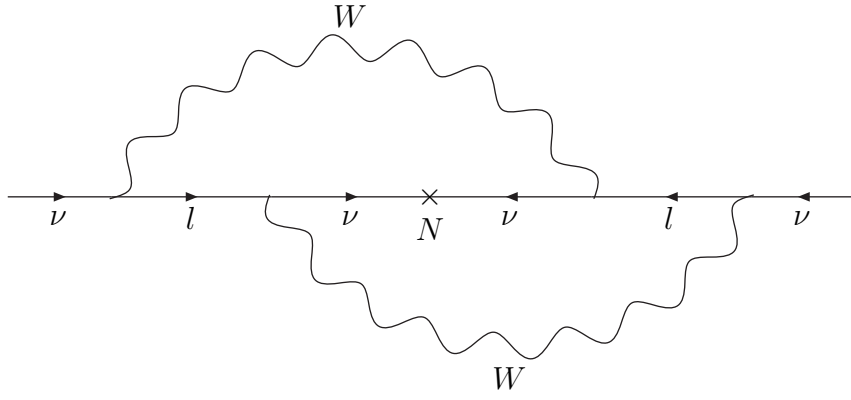
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**Fig. 1.** Two-loop radiative breaking of neutrino mass degeneracy.